# Communication in General Decentralized Filters and the Coordinated Search Strategy

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Abstract – This paper addresses the problem of coordinating multiple heterogeneous sensing platforms performing a search mission for a single target in a dynamic environment. In this approach, the decision makers build an equivalent representation of the Probability Density Function (PDF) of the target state by communicating with their neighbors in a decentralized Bayesian sensor network enabling them to coordinate their actions without exchanging any information about their plans. This paper focuses on the communication aspect of the network. A channel filter that handles general PDF's is developed. The channel filter assures that complete global information is recovered at each node provided that the network connectivity is acyclic. A channel manager is also developed that uses the Hellinger divergence measure between the node and channel estimates adaptively determines when to communicate on any particular channel. It is proven to significantly reduce the communication loads. Simulation results are used to evaluate the accuracy of the filtering algorithm and demonstrate the efficiency of the resulting coordinated search trajectories.

**Keywords:** general decentralized data fusion, channel filter, channel manager, coordinated control, search

# 1 Introduction

In an uncertain world, two questions arise when facing the problem of coordinating a team of heterogeneous sensing platforms involved in an information gathering task such as searching, surveying, exploring or tracking. First, how should all the information available about the task at hand be represented and fused with all the new sensor observations? Secondly, when and what should the team members communicate in order to increase their situation awareness but most importantly to coordinate their actions in an effective manner?

In [1], an active Bayesian sensor network approach to coordinating an arbitrary number of autonomous sensor platforms performing a search mission was introduced. The framework presented integrates a general decentralized Bayesian filtering technique with an adaptation of the decentralized coordinated control scheme first proposed by Grocholsky *et al.* [2]. In this approach, each sensor node builds an equivalent representation of the Probability Density Function (PDF) of the target state by exchanging observed information with all the other nodes in the Bayesian sensor network. Each decision maker locally plans based on that current common global picture of the world. By continuously exchanging information and dynamically altering and updating the prior on which these local decisions are made, the decision makers influence each other, rendering their trajectories globally consistent and coordinated. Scalability, modularity and real-time adaptability are the advantages of the decentralized approach.

One limitation of the technique as presented in [1] comes from the assumption that every sensor node transmits and receives every single observation without a miss via broadcasting. Beyond the obvious bandwidth limitations, such assumptions are not practical in real life since communication systems are plagued by delays and intermittent transmissions. This paper focuses on overcoming this problem. It presents an adaptation for general PDFs of the channel filter introduced in [3] which allows node-to-node communication in the Bayesian sensor network (Fig. 1). This significantly reduces the communication loads that are incurred in a fully connected network, as well as allowing intermittent burst communications and recovery from failed transmissions. The general distribution channel filter ensures that complete global information is recovered at each node provided that the network connectivity is acyclic, i.e. treeconnected without loops. The accuracy of the estimates is evaluated using the Hellinger affinity measure. A channel manager is also developed that uses the same divergence measure applied between the node and channel estimates to adaptively determine when to communicate on any particular channel.



Fig. 1: A general decentralized Bayesian sensor network with point-to-point communication.

The paper is organized as follows. First, Sec. 2 reviews the decentralized Bayesian filtering algorithm, that accurately maintains and updates the information about the target state, and the decentralized coordinated control strategy. Then Sec. 3 introduces the general channel filter and channel manager and discusses communication issues. Sec. 4 describes the searching problem. Sec. 5 evaluates the accuracy of the filtering algorithm and demonstrates the efficiency of the resulting coordinated search trajectories for a team of unmanned air vehicles (UAVs) searching for a single mobile non-evading target. Finally, conclusions and ongoing research directions are highlighted.

# 2 Active Bayesian Sensor Network

This section reviews the active Bayesian sensor network framework introduced in [1]. At the core of the approach is a general decentralized Bayesian filtering algorithm necessary to predict and update the highly non-Gaussian target state PDF. The Bayesian approach is particularly suitable for combining heterogeneous non-Gaussian sensor observations with other sources of quantitative and qualitative information [4, 5]. In this paper independent control laws are implemented at each sensor node hence making them active.

# 2.1 Bayesian Filtering

In the searching problem, the unknown variable of interest is the target state vector at time k, denoted  $\mathbf{x}_k^t \in \mathbb{R}^{n_x}$  which in general describes the target location but could also include its attitude, velocity, and other properties. In this paper the superscripts t and  $s_i$  indicate a relationship to the target and the sensor *i* respectively. The subscripts are used to indicate the time index. The purpose of the analysis is to find an estimate for  $p(\mathbf{x}_k^t | \mathbf{z}_{1:k})$ , the PDF over  $\mathbf{x}_k^t$  given the sequence  $\mathbf{z}_{1:k} = \{\mathbf{z}_{i}^{i} : i = 1, ..., N_{s}, j = 1, ..., k\}$ of all the observations made from the  $N_s$  sensors on board the search vehicles,  $\mathbf{z}_{j}^{i}$  being the observation from the  $i^{th}$ sensor at time step j. The analysis starts by determining a prior PDF  $p(\mathbf{x}_0^t | \mathbf{z}_0) \equiv p(\mathbf{x}_0^t)$  for the target state at time 0, given all available prior information including past experience and domain knowledge. If nothing is known other than initial bounds on the target state vector, then a least informative uniform PDF is used as the prior. Once the prior distribution has been established, the PDF at time step k,  $p(\mathbf{x}_k^t | \mathbf{z}_{1:k})$ , can be constructed recursively using the prediction and update equations alternatively.

# 2.1.1 Prediction

A prediction stage is necessary in Bayesian analysis when the target state PDF to be evaluated is evolving with time, i.e. the target is in motion or the uncertainty about its location is increasing. The target state transition model can generally be described by a set of time dependent non-linear difference equations as in

$$\mathbf{x}_{k+1}^t = \mathbf{f}_k^t(\mathbf{x}_k^t, \mathbf{u}_k^t, \mathbf{w}_k^t) \tag{1}$$

where  $\mathbf{w}_k^t$  is a system input vector that includes the process noise and the external forces acting on the system, and  $\mathbf{u}_k^t$ is the control input vector in the case of an active target. As the focus of this paper is on non-evading targets,  $\mathbf{w}_k^t$  will be the only system input discussed for the target.

Suppose the system is at time step k - 1 and the latest PDF update,  $p(\mathbf{x}_{k-1}^t | \mathbf{z}_{1:k-1})$ , is available. Then the predicted PDF of the target state at time step k is obtained from the following Chapman-Kolmogorov equation

$$p(\mathbf{x}_{k}^{t}|\mathbf{z}_{1:k-1}) = \int p(\mathbf{x}_{k}^{t}|\mathbf{x}_{k-1}^{t}) p(\mathbf{x}_{k-1}^{t}|\mathbf{z}_{1:k-1}) \, d\mathbf{x}_{k-1}^{t} \quad (2)$$

where  $p(\mathbf{x}_k^t | \mathbf{x}_{k-1}^t)$  is a probabilistic Markov motion or process model which maps the probability of transition from a given previous state  $\mathbf{x}_{k-1}^t$  to a destination state  $\mathbf{x}_k^t$  at time k. The process model is a function of the equations of motion for the target (1) and of the known distribution on their inputs,  $\mathbf{w}_k^t$ . Note that if the motion model is invariant over the target states, then the integral in (2) results in a convolution operation. Various examples of process models with constraints can be found in [6].

#### 2.1.2 Update

At time step k, a new set of observations  $\mathbf{z}_k = {\mathbf{z}_k^1, ..., \mathbf{z}_k^{N_s}}$  becomes available. For each sensor *i*, the mapping of the target state observation probability ,  $\mathbf{z}^i \in \mathbb{R}^{n_z}$ , for each given target state,  $\mathbf{x}_k^t \in \mathbb{R}^{n_x}$ , is referred to as the observation likelihood, or sensor model, and denoted  $p(\mathbf{z}_k^i | \mathbf{x}_k^t)$ . Assuming all the observations to be conditionally independent, the update for the prior PDF  $p(\mathbf{x}_k^t | \mathbf{z}_{1:k-1})$  (posterior from the prediction stage (2)) is performed using the following Bayes rule also referred to in the literature as the "independent opinion pool"

$$p(\mathbf{x}_k^t | \mathbf{z}_{1:k}) = K p(\mathbf{x}_k^t | \mathbf{z}_{1:k-1}) \prod_{i=1}^{N_s} p(\mathbf{z}_k^i | \mathbf{x}_k^t)$$
(3)

where the normalization coefficient K is given by

$$K = 1 / \int \left[ p(\mathbf{x}_k^t | \mathbf{z}_{1:k-1}) \prod_{i=1}^{N_s} p(\mathbf{z}_k^i | \mathbf{x}_k^t) \right] d\mathbf{x}_k^t$$
(4)

# 2.2 Control

Each of the  $N_s$  sensor/vehicle system is governed by its own dynamic model in the form

$$\mathbf{x}_{k+1}^{s_i} = \mathbf{f}_k^{s_i} (\mathbf{x}_k^{s_i}, \mathbf{u}_k^{s_i}, \mathbf{w}_k^{s_i})$$
(5)

where  $\mathbf{w}_k^{s_i}$  is the vector representing the process noise and the external forces acting on the system *i*, and where  $\mathbf{u}_k^{s_i}$ is the corresponding control input vector at time *k*. The controller objective is to produce a command that will place the system in a desired state.

#### 2.2.1 Optimal Trajectory

Optimality is defined in relation to an objective, or utility function [7]. For multiple sensor platforms, an optimal cooperative control solution must be a negotiated group decision that is jointly optimal.

For a control action sequence  $\mathbf{u} = {\mathbf{u}_1, ..., u_{N_k}}$ , with  $\mathbf{u}_j = {\mathbf{u}_j^{s_i} : i = 1, ..., N_s}$ , over a time horizon of length  $T = N_k \,\delta t$ , where  $N_k$  is the number of lookahead steps, the utility function is denoted  $J_k(\mathbf{u}, N_k)$  The optimal control policy  $\mathbf{u}^*$  is the sequence that maximizes that utility subject to the control bounds  $\mathbf{u}_{LB} \leq \mathbf{u} \leq \mathbf{u}_{UB}$  and the constraints  $\mathbf{g}(\mathbf{u}, N_k) \leq \mathbf{0}$ .

$$\mathbf{u}^* = \{\mathbf{u}_1^*, ..., \mathbf{u}_{N_k}^*\} = \arg\max_{\mathbf{u}} J_k(\mathbf{u}, N_k)$$
(6)

However, the computational cost for such optimal plans is subject to the "curse of dimensionality". With increasing lookahead depth and number of agents, the solution becomes intractable. In practice only solutions for very restricted number of lookahead steps are possible. Such optimal plans can only be obtained for a small number of agents and are out of reach in decentralized systems unless extensive negotiations occur between the agents. A solution for decentralized systems is to follow a decentralized coordinated control strategy [1].

#### 2.2.2 Decentralized Coordinated Control

A coordinated control solution is different from a cooperative control solution. In a coordinated control problem, decision makers plan individually based on their current knowledge of the world, i.e. target state PDF, and exchange information via the sensor network ensuring that each platform builds an equivalent representation of the target state PDF [2]. There is no mechanism to reach a negotiated outcome. Coordination results from the platforms affecting each other's local control decisions by contributing the prior on which these local decisions are made. For example, the utility for a vehicle to search a region with previously high probability density is decreased if another agent is already searching that region. This has the effect of increasing the relative utility of other regions of the space and diverting the former vehicle towards these regions. This explains how the vehicles avoid each other even though no collision avoidance system has been implemented.

Coordinated trajectories are suboptimal, but they have the following appealing advantages of being completely decentralized, computationally very cheap and highly scalable as the nodal planning computation costs do not increase with the number of platforms. As will be demonstrated in Sec. 5, the real-time adaptive plans are efficient and correspond to locally maximizing the individual payoff gradients. The simplest form of coordinated control is implemented with a lookahead depth of one-step corresponding to maximizing  $N_s$  independent control laws, i.e.  $J_k(\mathbf{u}_k^{s_i}, 1)$ for all sensor *i*. In this paper, the nodal greedy actions are obtained in real time using a constrained non-linear optimization technique called Sequential Quadratic Programming (SQP) [8].

# 2.3 Active Sensor Network

Packaging a physical sensor with its own Bayesian filtering processor is an attractive way of making the sensor mobile. Such a Bayesian sensor unit can be taken anywhere to take measurements about the world. Mounting the Bayesian sensor unto an actuated mobile Platform and coupling it to its own Controller makes it an active Bayesian sensor. Based on the latest belief about the world  $p(\mathbf{x}_{k-1}^t | \mathbf{z}_{k-1}^t)$  and the sensor state  $\mathbf{x}_{k-1}^{s_i}$ , the Controller sends a command  $\mathbf{u}_{k-1}^{s_i}$  to the Platform to place the sensor in a desired position  $\mathbf{x}_{k_{des}}^{s_i}$  with respect to the world to take the next observation. Fig. 2 depicts algorithmically the Bayesian fil-Sensor Node i



Fig. 2: General active Bayesian sensor node in a fully connected network with broadcast communications.

ter and how it interacts with the Controller, the Platform, and the sensor to form a node in a fully connected network. The Platform block represents the actuators and dynamics of both the sensor and the mobile vehicle, if present, on which the sensor is mounted. Any number of sensors can be attached to a particular fusion node. For simplicity in this paper, each sensor is packaged with its own node.

There is more than one valid way to implement the Bayesian filtering algorithm. For example, it is possible to represent the target PDF using parametric functions and to perform the prediction and update stage by updating the parameters of the function. If the target PDF as well as the process model are both Gaussian, then the most effective parametric filter is the well known Kalman filter. For the searching problem however, the process model, and especially the target PDF can be highly non-Gaussian and the complete description of the density function must be maintained. In this paper the prediction and update equations will be evaluated numerically using a grid based discrete approximation of the process model, the observation likelihood and the target PDF.

# **3** General Channel Filter

A limitation of the technique as presented in the above section comes from the assumption that every sensor node transmits and receives every single observation without a miss via broadcasting. Beyond the obvious bandwidth limitations, such assumptions are not practical as physical communication systems are plagued by delays and intermittent transmissions. To allow node-to-node communication in the network, it is desirable that the nodes communicate their posterior PDF instead of the their observation likelihood. This enables the individual node to extract and combine the information originating from beyond their immediate neighbors. In order to do so, the node of Fig. 2 must be modified by adding an extra estimator per communication channel as in Fig. 3a called a channel filter [3]. As will be seen, the channel filter ensures that complete global information is recovered at each node despite delayed communications in the network.

# 3.1 Node-to-Node Communication

At time step k, the incomplete set of observations available at node i is denoted  $\mathbf{z}_{1:k}^{*i}$ . The combined PDF estimate  $p(\mathbf{x}_k^t | \mathbf{z}_{1:k}^{*i} \cup \mathbf{z}_{1:k}^{*j})$  based on two incomplete but not mutually exclusive sets of observations,  $\mathbf{z}_{1:k}^{*i}$  and  $\mathbf{z}_{1:k}^{*j}$ , is obtained from

$$p(\mathbf{x}_{k}^{t}|\mathbf{z}_{1:k}^{*i} \cup \mathbf{z}_{1:k}^{*j}) \propto \frac{p(\mathbf{x}_{k}^{t}|\mathbf{z}_{1:k}^{*i})p(\mathbf{x}_{k}^{t}|\mathbf{z}_{1:k}^{*j})}{p(\mathbf{x}_{k}^{t}|\mathbf{z}_{1:k}^{*i} \cap \mathbf{z}_{1:k}^{*j})}$$
(7)

where  $p(\mathbf{x}_k^t | \mathbf{z}_{1:k}^{*i})$  and  $p(\mathbf{x}_k^t | \mathbf{z}_{1:k}^{*j})$  are the latest PDF estimates from nodes *i* and *j*, and  $p(\mathbf{x}_k^t | \mathbf{z}_{1:k}^{*i} \cap \mathbf{z}_{1:k}^{*j})$  is the estimate based on the common information. It is the purpose of the channel filter to maintain that common PDF estimate between two nodes so it can be removed (divided) from the product of the PDF's to be combined in order to prevent double-counting. The common PDF estimate will be referred to as the channel filter estimate. Eq. (7) appears in a similar form in [9] and an equivalent expression can be found in Sec. 4.10 of [4].

As illustrated in Fig. 3 the node's latest PDF is passed to the channel filter which divides it by it's own filter estimate to remove the common information. The residual which corresponds to the new information accumulated by the emitting node, through sensor observations and communication with other neighbors, is then communicated to the receiving Channel which uses it to update its own common estimate and passes it to the node. Once reception is acknowledged, the emitting Channel Filter also fuses the new information to update its own estimate. The Channel Filter



Fig. 3: General active Bayesian sensor node with node-to-node communication: (a) sensor node with its channel filters; (b) channel filter between two neighboring nodes.

is a recursive Bayesian filter which just like the node itself has a prediction step to account for the target motion and/or increasing uncertainty about its state. There can be multiple prediction/observation steps in between the communication steps.

One of the major advantage of implementing the Channel Filter comes from the fact that if for some reason a packet is lost in the communication process, the channel does not update its estimate allowing that information to be transmitted on the next communication step. Since the nodes only know their immediate neighbors and are ignorant of the global topology they cannot differentiate the source of the information they receive. One necessary condition to maintain proper accounting of the information is that the network connectivity must be acyclic [3]. In other words, no communication loops must exist between the nodes that would enable the information to cycle through multiple times.

#### 3.2 Issues with the General Channel Filter

The Channel Filter guarantees the nodes to converge exactly to the global estimate given a certain time delay, i.e. if there is no process involved. Otherwise small errors accumulate in the Node estimate as well as in the Channel Filter estimate during the prediction stage. This is attributed to the fact that there is incoming information in transit from other nodes still missing from the estimate during the prediction step. These errors are amplified with:

- the number of prediction steps and/or observations made between each communication step;
- the length of the communication chains in the networks;
- the rate of change of the PDF caused by the process, i.e. fast motion and/or diffusion rate;
- the amount of change in the PDF caused by any observation.

Notice that these errors in general produce conservative estimates in a decentralized filter [10] since the prediction forward for a set of observations before the update of the global PDF estimate reduces their impact on the estimate.

#### 3.2.1 Estimation Error

Estimation error accumulates in the nodal PDF estimates of the tree-connected with respect to the globally accurate fully connected network. To evaluate the amount of error, it is proposed to use the following alpha divergence measure, also known as the Renyi divergence

$$D(p_i \| p_j) = \frac{1}{\alpha - 1} \ln \int p_i(\mathbf{x}_k^t)^{\alpha} p_j(\mathbf{x}_k^t)^{1 - \alpha} d\mathbf{x}_k^t \quad (8)$$

which when  $\alpha = 1/2$  reduces to

$$D(p_i \| p_j) = 2 \ln \int \sqrt{p_i(\mathbf{x}_k^t) p_j(\mathbf{x}_k^t)} \, d\mathbf{x}_k^t \tag{9}$$

the Hellinger affinity measure, which as stated in [11] is monotonically related to a true distance metric between two densities  $p_i$  and  $p_j$ . The value range for the measure goes from 0, when  $p_i$  and  $p_j$  are the same, to  $-\infty$ , when the two PDF's have nothing in common, i.e.  $\int \sqrt{p_i(\mathbf{x}_k^t)p_j(\mathbf{x}_k^t)} d\mathbf{x}_k^t = 0.$ 

#### 3.3 Communication Management

In order to reduce the communication loads in a large network it is essential to implement some form of communication management to determine when to send information on each channel. In this paper it is proposed to implement a channel manager in each node which uses the Hellinger affinity measure (9) between the nodal PDF estimate and the channel filter estimate to determine the utility of communicating on a given channel at any given time.

Given a channel between two particular nodes, the algorithm works as follows. At initialization, the nodal estimates are the same as the channel estimate between the two nodes and the divergence measure is equal to zero. As time passes, the nodal PDF estimates diverge from the channel estimate by accumulating information from new sensor observations and/or from communication with their other neighbors. The channel manager determines that it is time to send the new accumulated information over the channel when the node and channel estimate are sufficiently different, i.e. when  $D(p_i||p_j)$  gets smaller than a certain threshold. Then both the channel and the receiving node estimates get updated.

Notice that in this architecture, the receiving node will not necessarily communicate back on the same time step, but it is most likely to pass on the newly received information onto its other neighbors as the boost in information, which suddenly increase the divergence measure with its the other channels, is likely to trigger more communications hops. The advantage of this approach is that each node determines on its own when it has accumulated enough new information to send to any particular network neighbor.

# 4 The Searching Problem

This section describes the equations for computing the probability of detection of a lost object referred to as the target by using the outputs of the prediction and update equations from Sec. 2.1. An equivalent but different derivation is presented in [1]. Further details on the searching problem can also be found in [12] and [13] (Chap.9).

Let the target detection likelihood (observation model) of the  $i^{th}$  sensor at time step k be given by  $p(\mathbf{z}_k^i = D_k^i | \mathbf{x}_k^t)$  where  $D_k^i$  represents a 'detection' event by sensor i at time k. The likelihood of 'no detection' by the same sensor is given by its complement  $p(\overline{D}_k^i | \mathbf{x}_k^t) = 1 - p(D_k^i | \mathbf{x}_k^t)$ . The combined 'no detection' likelihood for all the sensors at time step k is simply a multiplication of the individual 'no detection' likelihoods

$$p(\overline{D}_k | \mathbf{x}_k^t) = \prod_{i=1}^{N_s} p(\overline{D}_k^i | \mathbf{x}_k^t)$$
(10)

where  $\overline{D}_k = \overline{D}_k^1 \cap ... \cap \overline{D}_k^{N_s}$  represents the event of a 'no detection' observation by every sensor at time step k.

If the normalization factor K is neglected, the update equation (3) can be rewritten as

$$p(\mathbf{x}_{k}^{t}|\mathbf{z}_{1:k})' = p(\mathbf{x}_{k}^{t}|\mathbf{z}_{1:k-1})' \prod_{i=1}^{N_{s}} p(\mathbf{z}_{k}^{i}|\mathbf{x}_{k}^{t})$$
(11)

The advantage of not normalizing the target PDF at every update is that the joint probability of failing to detect the target in all of the steps from 1 to k, denoted  $Q_k = p(\overline{D}_{1:k})$ , can be directly obtained from the integration of the pseudo PDF update (11)

$$Q_{k} = \int p(\mathbf{x}_{k}^{t} | \overline{D}_{1:k})' d\mathbf{x}_{k}^{t} = \int p(\mathbf{x}_{k}^{t} | \overline{D}_{1:k-1})' p(\overline{D}_{k} | \mathbf{x}_{k}^{t}) d\mathbf{x}_{k}^{t}$$

where  $\overline{D}_{1:k}$  corresponds to the set of observations  $\mathbf{z}_{1:k}^{(12)}$ where every observation is a 'no detection', i.e.  $\mathbf{z}_k = \overline{D}_k, \forall k$ . Then, it can be shown that the probability the target gets detected for the first time on time step  $k, p_k$ , is given by the volume under the surface resulting from the product of the combined detection likelihood, denoted  $[1 - p(\overline{D}_k | \mathbf{x}_k^t)] = p(D_k | \mathbf{x}_k^t)$ , with the predicted target PDF, is equivalent to the reduction in volume  $(-\Delta Q_k)$  of the pseudo PDF as in

$$p_{k} = \int p(\mathbf{x}_{k}^{t} | \overline{D}_{1:k-1})' [1 - p(\overline{D}_{k} | \mathbf{x}_{k}^{t})] d\mathbf{x}_{k}^{t}$$
$$= Q_{k-1} - Q_{k}$$
(13)

Assuming no false detection from the sensors, the probability that the target *has* been detected in *k* steps, denoted  $P_k$ , is obtained from the cumulative sum of the  $p_k$ 's as in

$$P_k = \sum_{i=1}^{k} p_i = P_{k-1} + p_k \tag{14}$$

For this reason  $P_k$  will be referred to as the 'cumulative' probability of detection to distinguish it from the payoff probability of detection function  $p_k$ . Notice that plugging the expressions for  $p_k$  from (13) into (14) gives

$$P_k = 1 - Q_k \tag{15}$$

since  $Q_0 = \int p(\mathbf{x}_0^t) d\mathbf{x}_0^t = 1$ . This signifies that if the target PDF is not normalized after each update as in (11), then its volume,  $Q_k$ , represents the residual probability that the target is still present despite the search effort expended. Also, as k goes to infinity,  $Q_k$  decreases towards zero and  $P_k$  levels off towards one as it becomes harder to generate additional observation payoff,  $p_k$ , from hardly any probability mass left in the PDF.

As mentioned in [14], the goal of a searching strategy could be to maximize the chances of finding the target given a restricted amount of time by maximizing  $P_k$  over a given time horizon. For a time horizon of one as discussed in Sec. 2.2.2, the individual utility function reduces to  $J_k(\mathbf{u}_k^{s_i}, 1) = p_k^{s_i}$ , the probability of detecting the target on the next time step (13) which in turn is equivalent to the volume under the surface resulting from the product of the 'detection' likelihood from sensor *i* with the predicted pseudo target PDF,  $p(\mathbf{x}_k^t | \overline{D}_{1:k-1})'$ .

# 5 Application

Ultimately, the goal of the ongoing research effort is to demonstrate the coordinated decentralized search framework on a team of heterogeneous autonomous mobile platforms in various outdoor scenarios. A stepping stone towards this goal is to investigate the problem using simulation. The rest of this section presents the results from the decentralized coordinated search framework implemented for a team of 5 UAVs, such as illustrated in Fig. 4a, searhing a 4000 x 4000m area for a single lost target, a liferaft (Fig. 4b), drifting at sea. The motion model is a simple zero mean Gaussian diffusion process with a standard deviation in x and y of 35. More about the implementation details of the framework and the search problem can be found in [1].



Fig. 4: Search scenario: (a) The fleet of Brumby Mark-III developed at ACFR. These UAVs have a payload capacity of up to 13.5 kg and operational speed of 50 to 100 knots; (b) Search sensor aperture cone and geometrical relationship between the search vehicle and the target.

Fig. 6 illustrates the coordinated search results for the active Bayesian sensor network algorithm presented in Sec. 3 where the network nodes are connected in series such as shown in Fig. 5. The first five rows in the figure corre-



spond to the results where the channel manager communication threshold is set to  $D_{thresh} = -.02$ . Rows 1 to 3 represent the 3D views of the nodal PDF estimates and the corresponding vehicle trajectories for nodes N1, N3 and N5 respectively. The images represent snapshots taken at time step k = 100, 200 and 300 as well as the estimate after the four extra communication steps needed to reach full synchronization between the nodes. The fourth row shows all five vehicle trajectories and the exact broadcasted (BC) PDF estimate for comparison with the nodal estimates. Notice the small discrepancies between the PDF estimates due to the communications delays. The fifth row displays in order: the Hellinger divergence measures,  $D_k$ , representing the error in the nodal estimates with respect to the exact BC estimate. The second plot of the row compares the nodal and BC payoff functions  $p_k$ 's, while the third and fourth compares the cumulative probability of detection functions  $P_k$ 's.

The vertical gray line at step 300 on every plot represents the last simulation step after which the nodes simply communicate a few extra steps to ensure that all the information has reached all nodes. It can be seen on the  $D_k$  plot and confirmed on the  $P_k$  plot that even after the synchronization, the nodes display a little residual divergence from the broadcasted PDF due to the error accumulated in the channel filters. This error produce a conservative estimate reflected by the fact that the nodal cumulative probability estimates are always below the broadcasted  $P_k$ . The channel managers ensure that the communication intervals are adapted in such a way as that the nodal estimates do not diverge too far from the BC one as witnessed by the  $D_k$ and  $P_k$  plots. Clearly seen on the  $D_k$  plot is the reduction in nodal divergence that occurs during each communication burst. The communication steps can also be easily identified from the peaks in the  $p_k$  estimates and the steps in the  $P_k$  estimates. Also worth noticing on the zoom in of the  $P_k$  plot is that once a node communicates, it is usually followed soon after by the receiving nodes which transmit to their other neighbors and so forth until the new information that triggered the chain reaction is propagated throughout the network.

The two last rows in Fig. 6 correspond to the results where the channel manager communication threshold is set to  $D_{thresh} = -.005$ . For this case, the nodal PDF estimates are not presented as the differences between them and the exact BC one are hard to perceive visually. As can be seen on the divergence plot from row 7 when compared with row 5, reducing  $D_{thresh}$  to a quarter of the previous value has the effect of also reducing the nodal estimate error by a factor of four. Comparing the  $P_k$  plots shows a much tighter estimate of the cumulative probability of detection. This is achieved by more frequent communication bursts especially in the regions where a lot of probability of detection is accumulated as seen on the  $p_k$  and zoomed in  $P_k$  plots. The residual divergence left after the extra communication steps is also much smaller confirming that more frequent communications reduce that amount of accumulated error in the channel estimates. Also, because it affects the communication delays and hence the shape of the nodal PDF estimates, changing the value of  $D_{thresh}$  also affect the trajectories. Notice the stronger symmetry in the later case compared with the previous one.

Beyond having communication intervals adapted to the amount of new information entering the system, the channel manager has also the advantage of producing much more accurate nodal PDF estimates than a system with communications at fixed intervals. The reason for this is that in a fixed frequency system, an important amount of new information is not passed along to the other nodes right away on the following simulation steps, but rather on the next communication step. Hence, for a 5 node chain topology with a communication interval of 10, it would take 40 simulation steps (4 hops x 10 steps/interval) for new information to go from node 1 to node 5 and most likely only 4 steps with communication management. This results in lower nodal divergence during the experiment, but also much smaller residual divergence as much less error is accumulated in the channel estimates. The search platforms are also much less likely to interfere with each other since their PDF estimates are almost the same.

For evaluation of the channel manager performances, Figs. 7a to d show the resulting coordinated search results for a fully connected network with broadcasted communications at every time step. Provided that every node receives every observation from all the other nodes, no error accumulates in the PDF estimates. From the cumulative probability of detection plot in Fig. 7f it can be seen that when the channel communication threshold is small, i.e.  $D_{thresh} = -.005$  (case N2Nb on the plot),  $P_k$  closely follows the results from the fully connected (BC) case. The results diverge more when the threshold value is increased, i.e.  $D_{thresh} = -.02$  (case N2Na), as the resulting search trajectories are affected by the synchronization delays in the nodal PDF estimates. Nevertheless, for these two cases, the final cumulative probability of detection values,  $P_{300} = .877$  and .867 for case b and a respectively, are slightly better than for the fully connected network with  $P_{300} = .844$ . Finally, by allowing a more efficient allocation of the search effort, all of these three cases of decentralized coordinated search perform much better than the straight area coverage pattern shown on Fig. 7e with  $P_{300} = .719$ . In fact, the coordinated search with the small communication threshold did 22% better.

#### 6 Summary and Ongoing Work

This paper addressed the problem of coordinating multiple, possibly heterogeneous, sensing platforms performing a search mission for a single target in a dynamic environment. However, the method is readily applicable to searching problems of all kinds, let it be on land, underwater, or



Fig. 6: Coordinated search results with node-to-node communication and channel manager: (row 1 to 5) results for channel communication threshold set to  $D_{thresh} = -.02$ , where (row 1 to 3) are the 3D views of the nodal PDF estimates and the corresponding vehicle trajectories at time step k = 100, 200, 300 and after synchronization for node  $N_1$ ,  $N_3$  and  $N_5$  respectively, and where (row 4) are same trajectories overlaid with the 3D views of the exact PDF estimate evolution; (row 6,7) results for  $D_{thresh} = -.005$ , where (row 6) are the vehicle trajectories overlaid with the 3D views of the exact PDF estimate evolution.



Fig. 7: Coordinated search with broadcast communication: (a) to (d) 3D views of the target PDF and the coordinated trajectories evolution at time step k = 1, 100, 200 and 300 respectively; (e) Straight pattern search at k = 300, and (f)  $P_k$  vs. k for the broadcast, node-to-node (case a and b), and flight formation cases.

an airborne search for bushfires, lost hikers, enemy troops in the battlefield, or prospection for ore and oil, or even to search for water or evidence of life on another planet.

The general decentralized Bayesian framework presented was demonstrated to adaptively find efficient coordinated search plans that explicitly considers the search vehicles kinematics, the sensors detection function, as well as the target arbitrary motion model. Coordinated solutions are suboptimal, but they have the appealing advantages of being adaptive and completely decentralized. As such, because nodal computation costs are kept constant with the number of platforms, they offer tremendous scalability potential limited only by the bandwidth of the communication medium.

A channel filter that handles general probability density functions was developed to allow node-to-node intermittent burst communications and enable recovery from lost packets and transmission delays which plague practical communication systems. A channel manager that significantly reduces the communication loads was also developed. To evaluate the accuracy of the resulting nodal PDF estimates, it was proposed to use the Hellinger divergence measure. The same measure was also used with great effect by the channel manager to adaptively determine when to communicate on any particular channel.

As part of the ongoing research effort, techniques such as Monte Carlo methods, or particle filters [15], as well as the so called kernel methods for density estimation [16] are being investigated to overcome the "curse of dimensionality" limitations of the grid based approach presented. Techniques to facilitate human interactions with the active Bayesian network, and a Negotiation Filter to increase the time horizon of the decentralized search plans are also being investigated.

Beyond the demonstration of the approach on a team of UAV's, the ultimate objective of this research is to eventually have multiple platforms participating in actual search and rescue missions with real-time cooperative planning and fully integrated human inputs in the loop. As shown by the results presented, this technique has the potential to greatly improve upon current search and rescue protocols, which in turn could be critical in saving human lives.

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#### References

- F. Bourgault, T. Furukawa, and H.F. Durrant-Whyte. Coordinated decentralized search for a lost target in a Bayesian world. In *IEEE/RSJ Int. Conf. on Intelligent Robots and Systems (IROS'03)*, October 2003.
- [2] B. Grocholsky, A. Makarenko, and H. Durrant-Whyte. Information-theoretic coordinated control of multiple sensor platforms. In *IEEE Int. Conf. on Robotics and Automation (ICRA'03)*, 2003.
- [3] S. Grime and H.F. Durrant-Whyte. Communication in decentralized systems. IFAC Control Engineering Practice, 2(5):849–863, 1994.
- [4] J.O. Berger. Statistical decision theory and Bayesian analysis. Springer series in statistics. Springer-Verlag, New York, 2nd edition, 1985.
- [5] L.D. Stone, C.A. Barlow, and T.L Corwin. *Bayesian Multiple Target Tracking*. Mathematics in Science and Engineering. Artech House, Boston, 1999.
- [6] F. Bourgault and H.F. Durrant-Whyte. Collaborative multi-robot exploration. In *IEEE Int. Conf. on Robotics and Automation (ICRA'04)*, New Orleans, USA, April 2004.
- [7] K.L. Teo, C.J. Goh, and K.H. Wong. A Unified Computational Approach to Optimal Control Problems. Longman Scientific and Technical, 1991.
- [8] G.L. Nemhauser, A.H.G. Rinnooy Kan, and M.J. Todd. Vol. 1: Optimization. Handbooks in Operations Management Science. Elsevier Science Publishers B.V., Amsterdam, 1989.
- [9] M.E. Liggins, C.Y. Chong, I. Kadar, M.G. Alford, V. Vannicola, and S. Thomopoulos. Distributed fusion architectures and algorithms for target tracking. In *IEEE Special Issue on Sensor Fusion*, volume 85, pages 95–107, January 1997.
- [10] E. Nettleton and H. Durrant-Whyte. Delayed and asequent data in decentralised sensing networks. In G.T. McKee and P.S. Schenker, editors, *Sensor Fusion* and Decentralised Control in Robotic Systems IV, volume Proc. SPIE, pages 1–9, 2001.
- [11] A.O. Hero, B. Ma, O. Michel, and J. Gorman. Alpha-Divergence for Classification, Indexing and Retreival (Revised 2). Technical Report CSPL-328, Communication and Signal Processing Laboratory, The University of Michigan, 48109-2120, USA, June 2002.
- [12] L.D. Stone. Theory of Optimal Search, volume 118 of Mathematics in Science and Engineering. Academic Press, New York, 1975.
- [13] J.S. Przemieniecki. Mathematical Methods in Defense Analyses. AIAA Education Series. American Institute of Aeronautics and Astronautics, Inc., Washington, DC, 2nd edition, 1994.
- [14] F. Bourgault, T. Furukawa, and H.F. Durrant-Whyte. Optimal search for a lost target in a Bayesian world. In *Int. Conf. on Field and Service Robotics* (FSR'03), July 2003.
- [15] N.J. Gordon, D.J. Salmond, and A.F.M. Smith. Novel approach to nonlinear/non-Gaussian Bayesian state estimation. *IEE Proceedings-F*, 140(2):107–113, April 1993.
- [16] B.W. Silverman. Density Estimation for Statistics and Data Analysis. Monographs on statistics and applied probability; 26. Chapman and Hall, London; New York, 1986.